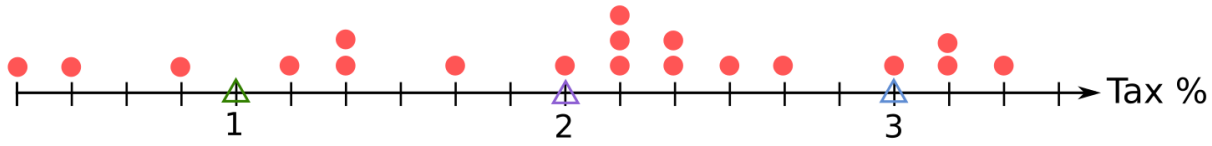
CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Spatial and Hedonic models
Bhaskar Kumawat

Question 1: Cut points

Consider the following diagram. Each point (filled circle) on the axis denotes a single individual voter. Each voter has an opinion about how much tax should be levied on the rich (tick marks go from 5% to 100% with step size 5%). There are three candidates (open triangles), each of whom promise different amounts of taxes (in percent) for the rich if they are elected.



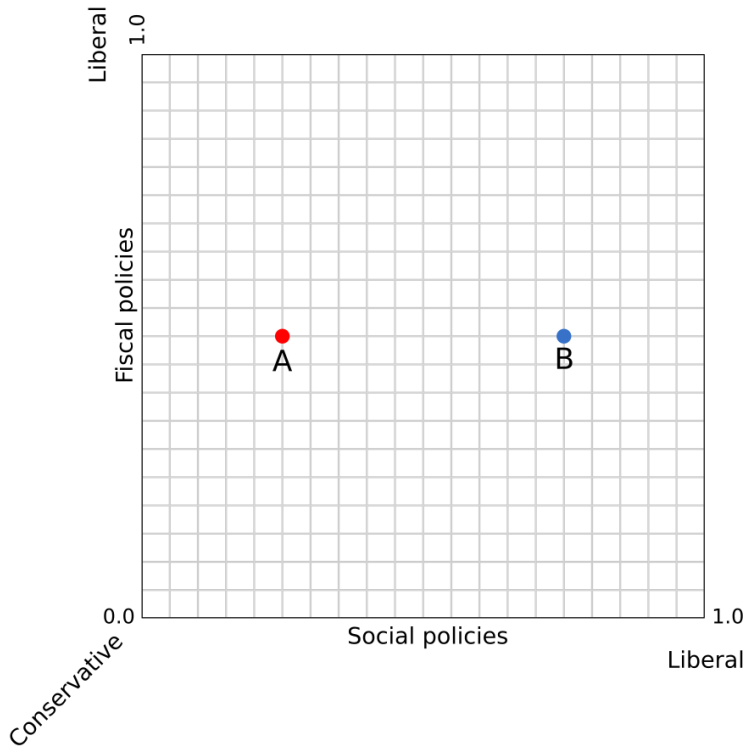
1. Assuming each voter votes for a candidate that promises the closest tax percent to their opinion, what are the number of voters that vote for each candidate (assume any voters on the borderline vote for the higher taxing candidate)? Who wins in this case?

2. The voters on the above line vote for different candidates on different sides of two "cut-points". These two cut points divide the entire axis into three regions, with each region denoting the region of favor for each candidate. What are the cut-points (in %) on the above line?

3. Based on these cut-points, we can define a "region of favorability" for each candidate, which is just the size of the region of x-axis for which the candidate is the preferred choice. What are the sizes of the regions of favorability for each candidate?

Question 2: Cut lines

Now consider a case where there are multiple parameters that can define the favorability of a candidate. In this case, let this be the nature of their fiscal and social policies (rated between 0 and 1 from conservative to liberal). Let us say there are two candidates that have the same fiscal policy (value 0.5) but differ in their social policies (0.25 and 0.75 respectively, see diagram). Once again, assume that a voter votes for the candidate whose social policy is closest to their own ideology (in terms of distance in the diagram).



1. Shade the region in the above diagram where a voter (if present in that region) votes for the more conservative candidate. This is the region of favorability for the conservative candidate. Compare the sizes of the regions of favorability for the two candidates.

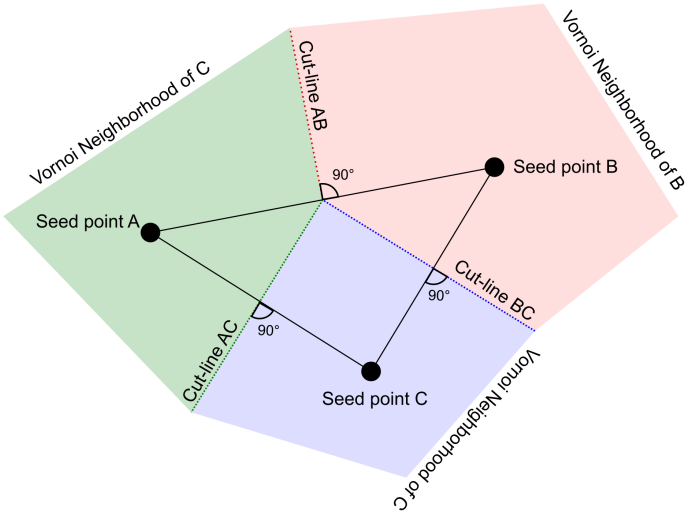
2. The line separating these two regions is called the "cut-line". Say you are a third candidate who wants to stand for the election but you don't want any of the other candidates to have a clear majority over the other (i.e., the relative sizes of their regions of favorability remains the same). What is the nature of fiscal/social policies you'll adopt to ensure this?

2. In the previous question, you have made a subtle assumption about whether a candidate wins based on the size of their region of favorability. What is this assumption? (i.e., in what case does the size of the region of favorability determine majority?)

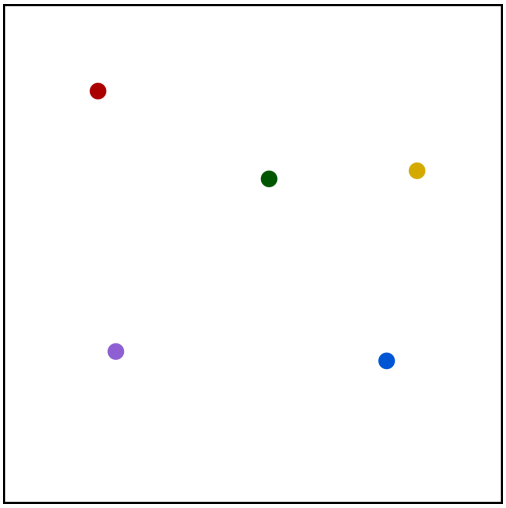
Question 3: Vornoi Neighborhoods

The regions bounded by the cut-lines in the previous question (or the "regions of favorability") are called **Vornoi neighborhoods** of their respective "seed" points (the candidates). Each point in the Vornoi neighborhood of a seed point is closest to that seed point than the other seed points. Here's a geometric fact about the cut-lines dividing different Vornoi neighborhoods.

The cut-line separating the Vornoi neighborhoods for two points is the perpendicular bisector of the lines connecting the two seed points. (See diagram below)



1. Using the above geometric fact, draw the approximate Vornoi neighborhoods for a set of five political candidates as shown in the figure below.



1. Now assume there's a third parameter that *always* makes a candidate more favorable to the voters (for example, the amount of money their campaign spends on publicity). A candidate that spends more on their campaign thus essentially expands their cut-line by a value proportional to this amount.

You are a donor that wants to donate money to the candidate that will gain the maximum new voters with a given amount. Which of the above five candidates should you donate your money to? Why? (Think about how the area of the Vornoi neighborhood changes when you expand the cut-lines outwards)

Discussion Feedback (anonymous; though you'll need to login to your UM google account to prevent unauthorized access): `https://forms.gle/WyP7o66k6hnozcHA8`