CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Game Theory and collective action
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Question 1: Dominant strategies and Nash Equilibria

Consider a prisoner's dilemma game where there are two prisoners 1 and 2, and each of them are given the choice to either cooperate (C) or not cooperate (NC) during an interrogation. Depending on both prisoner's choices during the interrogation, they are given a particular length of prison sentence (in years). To model them as rational agents trying to maximize utility, we denote the "utility" of a sentence as the negative of its length (in years).

1. Consider the following utility table and assume that you are player 1. Which of the two strategies (C or NC) should you choose in order to minimize your sentence (i.e., maximize utility) if you don't know what the other player is going to choose? Why?

		Prisoner 2	
		Cooperate (C)	Not Cooperate (NC)
Prisoner 1	C	-2 , -2 (Player 1 utility, Player 2 utility)	-10 , -1
	NC	-1 , -10	-5 , -5

A strategy that has a better utility in all cases, irrespective of what the other prisoner chooses, is called a **dominant strategy**. In the above case, your answer (C or NC) is a dominant strategy. Unfortunately, most games that we encounter are unlikely to have a clear dominant strategies. Instead, we are more likely to find something called a Nash equilibrium (Named after John Nash, the winner of a Nobel Prize in Economics and also portrayed famously by Russell Crow in the 2001 movie *A Beautiful Mind*).

Now, consider the table below.

		Player 2	
		A	B
Player 1	A	2, 1 (Player 1 utility, Player 2 utility)	0, 0
	B	0, 0	1, 2

2. Assuming you are player 1, is there a dominant strategy you can choose? If so, which one is it (A or B)?

3. A **strategy profile** is an ordered set of strategies chosen by each player in a game. For example, (A, B) is a strategy profile for the case when player 1 chooses strategy A and player 2 chooses strategy B. Each possible strategy profile in a game has a utility for each player. For example, the utility for player 1 when the strategy profile is (A, B) is denoted by $u_1(A, B)$. In the space below, write down the utilities for each player for each possible strategy profile (using this notation). One is given as an example.

$u_1(A, A) = 2$

4. The concept of **Nash equilibrium** applies to strategy profiles and not individual strategies. A strategy profile (s_1, s_2) is a Nash equilibrium if.

$$u_1(s_1, s_2) \geq u_1(s_1', s_2), \text{ and}$$
$$u_2(s_1, s_2) \geq u_2(s_1, s_2')$$

Where s_1' and s_2' are alternate strategies that the players can choose. (This might be a bit tricky to understand. Take some time to discuss the notation with your group. You can also ask me for help!)

What are the Nash equilibrium strategy profiles in the above game?

5. Based on the definition above, and your answer, can you write a rough definition of a Nash equilibrium in your own words?

Question 2: Collective action

Consider a group of N people that each has an income of I (greater than N). A person can invest some of this income into public goods and use the rest for their own private spending. Let us say the utility function for each individual is given by,

$$\text{Utility} = 2\sqrt{\text{PUBLIC}} + \text{PRIVATE}$$

Where PUBLIC is the total amount spent on public goods by everyone in the group, and PRIVATE is the amount spent by each for their own benefit.

1. If everyone uses an amount X from their income on public goods and the rest for private use, write the utility for every person in the group in terms of I and X. What is the total group utility?

2. The group utility function has two terms that vary with X. What can you say about the shape of the graph of these terms when plotted against X? (i.e., are they linear, superlinear, or sublinear)?

Based on what we know about shape of curves, the total utility actually peaks at a particular value of X. This value is in fact, exactly equal to N. So, to maximize the group benefit, each person must contribute an amount equal to the size of the group!

3. Now assume that a particular individual contributes an amount X to the public goods but everyone else contributes an amount A. Write the utility function for this individual (in terms of A and X). Hint: If this person contributes X, there are only $N - 1$ other players that can contribute to the public goods.

This function also peaks at a particular value of X which is exactly equal to $1/N$ (if all players contribute the same amount). Thus, to maximize individual utility, a person must contribute an amount which is inversely proportional to the size of the group!

4. For what value of N are the group and the individual utilities maximized simultaneously?

Discussion Feedback (anonymous; though you'll need to login to your UM google account to prevent unauthorized access): `https://forms.gle/WyP7o66k6hnozcHA8`