CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Individual value in groups
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Question 1: Last-on-the-bus value (homogeneous case)

You are a private business owner looking for consultants to help your business grow. Based on your projections, a consulting team consisting of three business majors (Anna, Bob, and Carol) can allow you to increase your profits by 1500$. On the other hand, if you employ any two of these people for the job, you'll only increase profits by 1000$.

1. Write down the increase in profit you'll get by employing only two business majors (for each possible pair).

2. Now let us assume you decide to employ all three of them for a month, thus making an extra 1500$ worth of profit that month. Following that, you decide to lay off one of these business majors. How much **decrease in profits** will you suffer in the next month if you decide to lay off Anna, Bob, or Carol (This might be redundant, but write the answer for each).

The amounts that you've given as answers to question 2 are what is referred to as the **Last-on-the-bus (LOTB) value** for each person. It measures the relative cost of removing (or benefit of adding) a person to a group assuming the rest of the group has already formed.

3. Let us say you lay-off Carol in the previous question. This leaves you with just two consultants, Anna and Bob. As you still have a decent increase in profits, you decide to be greedy and remove another consultant from the team, not knowing that a single consultant will help you increase monthly profits only by 300$. What is the LOTB value for Anna and Bob in this case?

4. Is there a diminishing or increasing returns to adding more consultants to the team? Why?

Question 2: Last-on-the-bus value (inhomogeneous case)

In the last case, the people being employed were essentially homogeneous and had no unique skills. The case becomes slightly more complicated when the group is more diverse. Consider the following list of increases in profit you can attain by employing certain combinations of the consultants.

Group	Increase in profit (Group value)
Anna	100$
Bob	80$
Carol	50$
Anna & Bob	600$
Anna & Carol	800$
Bob & Carol	1000$
Anna, Bob, & Carol	1500$

1. Based on the above table, what are the LOTB values for Anna, Bob, and Carol assuming you are making a group of 3 consultants?

2. Compare the LOTB values for each consultant to their values when they work alone. What does this tell you about the benefit of group work in this case?

3. Does the LOTB values for each player add up to the total value of having three consultants?

3. What are the LOTB values for each person if you are making a group of 2 consultants?

Question 4: Shapley value

The last-on-the-bus value assumes that there is an ideal group size and only one individual is removed at a time to judge their utility. However, it is possible that individuals have better or worse marginal value in smaller groups (see last question for example, where LOTB values are different across group sizes).

A more useful measure in this case is the **Shapley value** of individuals. Here's a slightly formal definition: "The Shapley value of an individual is their LOTB value averaged across all orderings, assuming they are the last to join." Let's try to calculate the Shapley values for Anna, Bob, and Carol in the above example, step-by-step:

1. Write down all possible ways in which you can arrange the group of three - Anna, Bob, and Carol (You can use ABC instead of their full names).

2. Let's calculate the Shapley value for **Anna**. Write the above orderings below, assuming Anna is the last to join the group (i.e., rewrite the orderings after removing anyone who joined after Anna).

3. Using the table from Question 2, calculate the LOTB value for Anna in each of the above orderings.

4. The Shapley value for Anna is just the average of the numbers above. What is this value?

You can similarly calculate the Shapley values for both Bob and Carol as well. Here are three interesting facts about Shapley values.

1. The sum of Shapley values for all individuals in a group is the value of the entire group.
2. The Shapley value of an individual approximates their contribution to the group in every possible scenario.

3. The Shapley value for each individual in a group might be different from the values when they work alone. In fact, Carol's Shapley value in this example is the highest, even though they have the smallest individual value.

5. **Bonus HW Question (2 extra points, due today at 11:59pm in the "In-class discussion" link on Canvas):** Calculate the Shapley values for Bob and Carol and verify that the sum of individual Shapley values is equal to the group value (show your work).

Discussion Feedback (anonymous; though you'll need to login to your UM google account to prevent unauthorized access): `https://forms.gle/WyP7o66k6hnozcHA8`