CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Optimization landscapes
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Question 1: Hill climbing with a single global optimum

Every week, you buy a scratch card from a lotto machine that has a serial number in the form of a letter (between A-L) and a number (between 1-12). Each serial numbered scratch card reveals a prize (in dollars) associated with it and this prize is dependent on the serial number on the card (For example, a scratch card numbered A11 will always reveal a prize of 20$).

Using this numbering and the associated prize we can make a"fitness landscape" for the lotto tickets as follows (the number in a cell is the reward associated with a ticket having that serial number).

Fitness Landscape 1:

	1	2	3	4	5	6	7	8	9	10	11	12
A	35	40	45	50	55	75	80	85	90	85	80	75
B	40	45	50	55	75	80	85	95	95	95	85	80
C	40	45	50	55	75	80	90	95	100	95	90	80
D	40	45	50	55	75	80	85	95	95	95	85	80
E	35	40	45	50	55	75	80	85	90	85	80	75
F	30	35	40	45	50	55	75	80	80	80	75	55
G	25	30	35	40	45	50	55	75	75	75	55	50
H	20	25	30	35	40	45	50	55	55	55	50	45
I	15	20	25	30	35	40	45	50	50	50	45	40
J	10	15	20	25	30	35	40	45	45	45	40	35
K	5	10	15	20	25	30	35	40	40	40	35	30
L	0	5	10	15	20	25	30	35	35	35	30	25

1. When you buy a ticket for the first time, you get a serial number randomly from one of the 12x12 (144) possibilities. Use this link (`https://g.co/kgs/imEVvJ`) to generate **two** separate random numbers between 1 and 12 (Make sure you set min=1 and max=12). Your initial scratch card number is the first random number as a letter + the second random number. For example if you got (8,11) as the two random numbers, your initial serial number is H11. Write your initial serial number in the space below and also shade the cell in the above diagram.

2. You scratch the initial card and find out you win whatever amount is associated with the serial number on the scratch card (this is the only prize you know on the fitness landscape for now). Being a careful fellow, you decide to explore the landscape in a systematic manner using a rule.

Rule 1:

1. Denote the serial number for the card I bought as the "home card".
2. In the next week, buy all four neighbors of the home card. These are the cards you get by changing either the letter or the number by 1 (but never both). In the above grid, for example, D3, C4, D5, and E4 are neighbors of D4.
3. If any of these cards has a higher prize than the current home card,
   - Denote the highest prized serial number as the new home card. If two of them have the same prize value (that is higher than the home card), choose one of them randomly to be the new home card.
   - Repeat from step 1 for the new home card.
4. If none of the newly bought cards have a higher prize, keep the current home card and stop buying new cards.

2. Using the above rule, the fitness landscape on the first page, and the initial serial number you picked - trace the set of home-cards you will have till you stop buying new cards. What is the home-card you end up with? How many weeks did it take you to get to this home-card?

You will find that every student in the class will end up with the same home-card! This is the special case where there is only one "peak" in the landscape. The set of rules above is a "hill-climbing" algorithm that prioritizes increase in utility at every step.

Question 2: Hill climbing on a slightly rugged landscape

1. Generate a random starting position on the landscape once again. Trace the home-card serial numbers that you will get over time until you stop buying new ones on the following landscape. What is your final home-card serial number?

	1	2	3	4	5	6	7	8	9	10	11	12
A	35	40	45	50	55	75	80	85	90	85	80	75
B	40	45	50	55	75	80	85	95	95	95	85	80
C	40	45	50	55	75	80	90	95	100	95	90	80
D	40	45	50	55	75	80	85	95	95	95	85	80
E	35	40	45	50	55	75	80	85	90	85	80	75
F	30	35	40	45	50	55	75	80	80	80	75	55
G	25	30	35	50	45	50	55	75	75	75	55	50
H	20	25	50	55	50	45	50	55	55	55	50	45
I	15	50	55	60	55	50	45	50	50	50	45	40
J	10	15	50	55	50	35	40	45	45	45	40	35
K	5	10	15	35	25	30	35	40	40	40	35	30
L	0	5	10	15	20	25	30	35	35	35	30	25

Question 3: Hill climbing on a very rugged landscape

Generate another random starting position on the landscape. Trace the home-card serial numbers that you will get over time until you stop buying new ones on the following landscape. What is your final home-card serial number?

	1	2	3	4	5	6	7	8	9	10	11	12
A	65	40	45	70	55	30	80	85	90	45	50	45
B	40	45	15	55	55	60	55	95	90	95	45	80
C	40	5	15	20	20	55	55	90	100	95	90	80
D	40	45	20	55	75	55	60	50	95	20	5	20
E	45	40	45	50	55	60	65	55	60	5	10	5
F	30	60	40	45	50	55	60	55	55	20	5	20
G	25	30	35	50	45	50	55	75	75	75	55	50
H	20	25	50	55	50	45	50	55	55	55	50	45
I	15	50	55	60	55	50	45	50	50	65	45	40
J	0	0	50	55	50	35	40	45	45	80	65	35
K	5	10	0	35	25	30	45	70	40	65	35	30
L	20	5	0	15	20	45	70	85	70	35	30	25

Compare your answers with those of your classmates. What fraction of the class ends up in the most optimal serial number (i.e., the one with the maximum prize) in each of the three questions above?

Question 4: In a small group, brainstorm a way to create a rule that will always take you to the global optimum even in a very rugged landscape. You don't need to come up with a perfect one. Just come up with something that might work and try it out with the landscape in Q3. Write a rough description of your rule below (Note: Buying all the cards is not a valid answer! Let's say you only earn enough to buy four scratch cards a week.)

Discussion Feedback (anonymous; though you'll need to login to your UM google account to prevent unauthorized access): `https://forms.gle/WyP7o66k6hnozcHA8`