CMPLXSYS/POLSCI 391 — Winter 2022 — Discussion
Linear and Non-linear models
Bhaskar Kumawat

Group names: ______________________________

Model type: ______________________________

Two variable relationships

A lot of processes in the real world can be represented as a set of points on a two-dimensonal plane - consisting of X and Y axes. One of these variables is independent (usually represented on the x-axis) and the other is dependent (on the independent variable; represented on the y-axis). A few examples include,

- Number of disease infected people (dependent) as a function of time (independent)
- A person's political alignment (dependent) as a function of their latitude (independent).
- The output of an economy (dependent) as a function of total investment (independent).

These relationships are usually fit to a model to extrapolate and predict things in the future. In most cases, a look at the data gives a good idea of the type of model we should use. For example, in figure 1 (next page) it is very clear what type of model you should use for the data.

Sometimes, however, it is not possible to guess the model you should use just from the shape of the data. In even more extreme cases, the data might look completely different from what we predict from theory. This might indicate a flaw or deviation from the assumptions we're using for our model. In that case, what we need to do is figure out the type of processes that might be creating the pattern. Certain processes are likely to give superlinear (convex) growth whereas others might be sublinear (concave) or linear.

In this discussion we'll split into groups and try to reason out why some relationships might be linear or non-linear in nature.

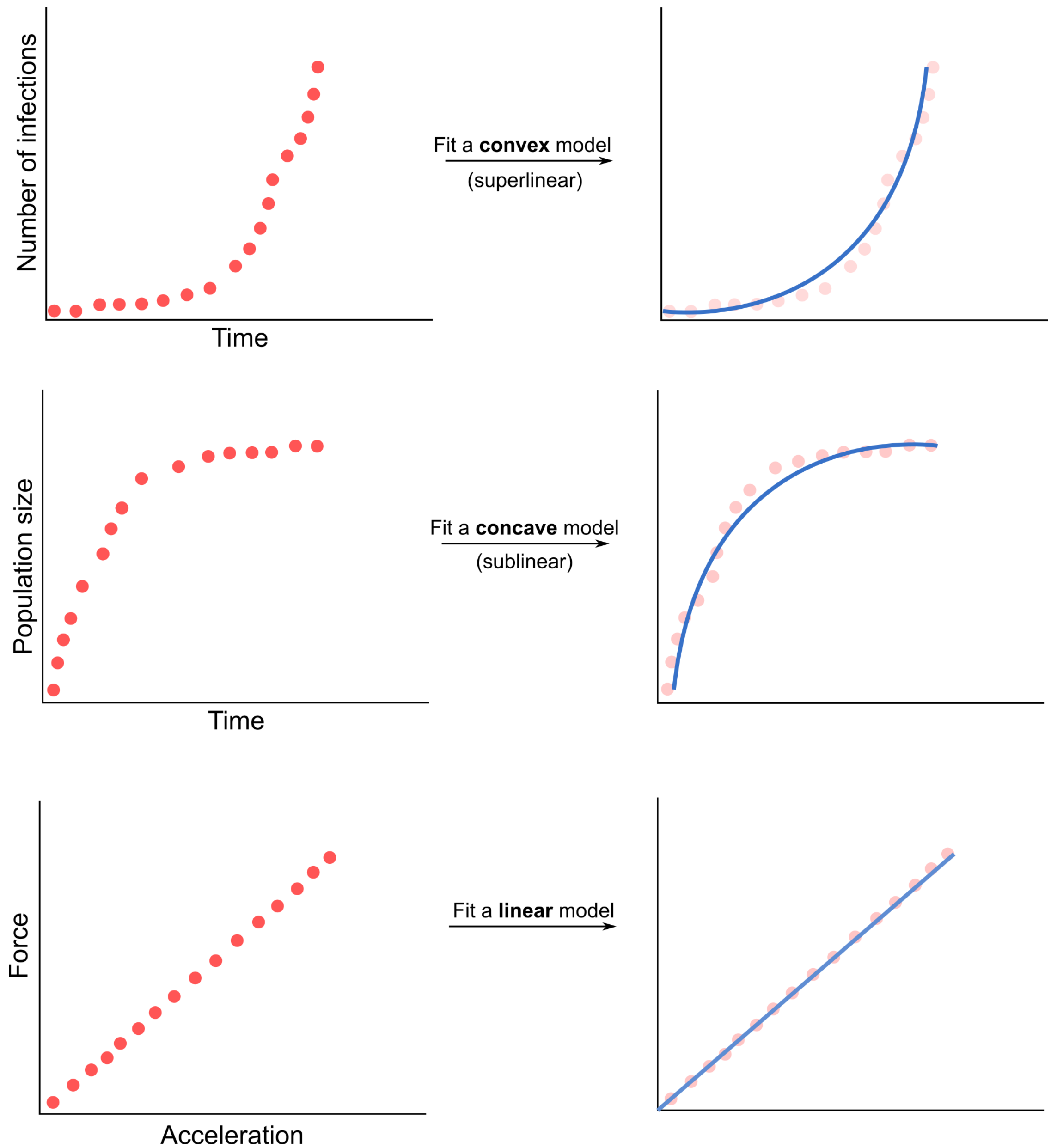

Figure 1: Data and the corresponding best-fit models.

Group discussion prompts

Your GSI will assign you one of either linear/superlinear/sublinear models. All the subsequent prompts are for your model type only.

1. The slope is the slant of a curve in the x-y plane with respect to the axis. The slope at a point is defined as the angle that a tangent to the curve at the point makes with the x-axis (See figure below).

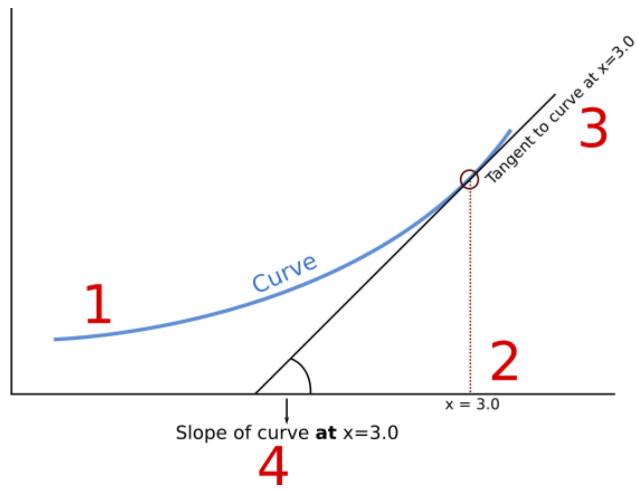

Figure 2: Slope of a curve at a given point as the slope of the tangent. The numbers indicate the step-by-step process for calculating the slope.

In the case of your model type (linear/superlinear/sublinear), what happens to the slope as you move rightward on the x-axis?

2. Think of some examples of things that increase linearly/superlinearly/sublinearly (with respect to either time or any other variable). These examples can be as real or as constructed as you want. Given below are some keywords to stimulate your thinking in this direction.

- **Linear:** Constant rate, uniform increase, rate of change for very small values is same as that for very large values
- **Superlinear:** Exponential, increasing rate, explosive increase
- **Sublinear:** Diminishing returns, saturation, decreasing rate

3. What are some elements of commonality between these processes?

4. Say that you have a set of nodes, each representing a person that can either be infected (empty circle) or not-infected (filled circle). You start with one infected person in the population. Edges between agents indicate interactions that lead to spread of infection. At each time point the infection spreads, infecting every person connected by a single edge to an infected individual (see example below).

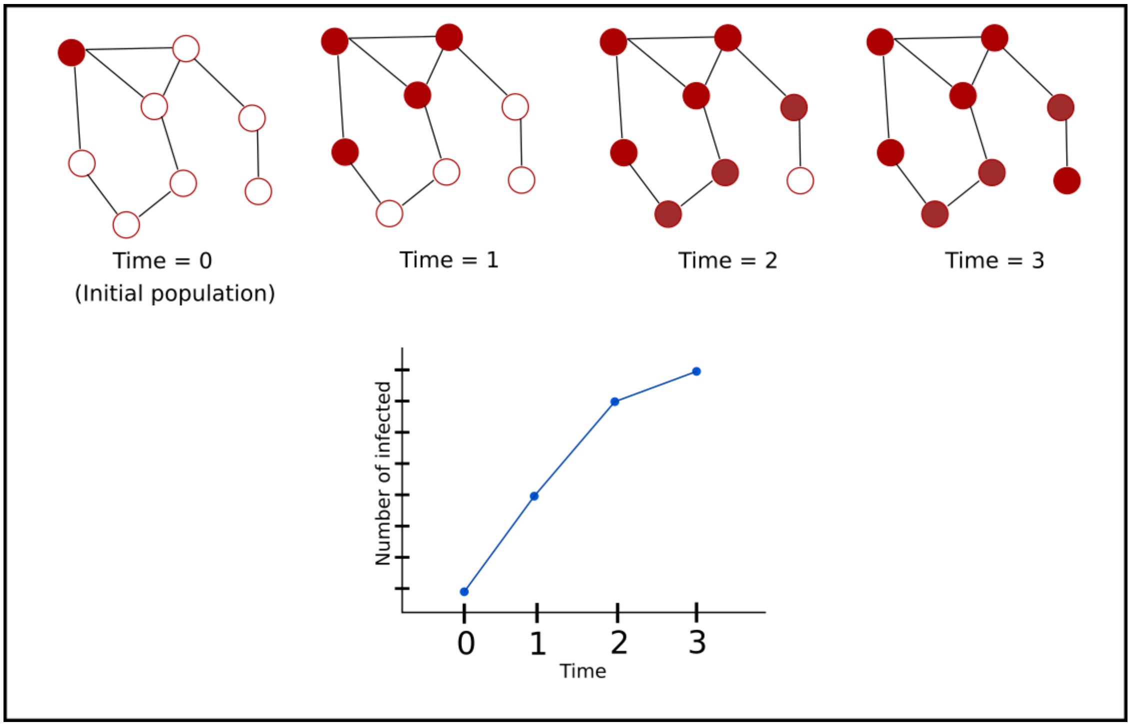

Figure 3: Model of infection in a network of individuals

You will be given a schematic of a set of individuals that are not connected to each other but there is a patient zero that starts the infection. Your goal is to, as a team, figure out any network structure that gives a linear/superlinear/sublinear increase in the number of infected over time. Use ideas from the previous prompts to figure out a way to do this. **Tip**: To track this, one of your team members can write down the number of infected at every time step in a table on another sheet of paper or in excel (you can use a pencil to fill out the empty circles and see how the infection proceeds over time).

Verify the linearity/superlinearity/sublinearity of the process by drawing a graph of the number of infected against time.

Have a look at some of the networks from the other teams in the class. What characteristics of the networks differ for the three kinds of processes?

Discussion Feedback (anonymous; though you'll need to login to your UM google account to prevent unauthorized access): `https://forms.gle/WyP7o66k6hnozcHA8`